
The Nash-Tognoli theorem over the rational numbers and its version for isolated singularities

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Abstract

In this I talk will introduce and discuss several possible approaches to addressing the following open question:

Q-algebraicity problem: (Parusinski, 2021) *Is every algebraic subset X of $\mathbb{R}^n$ homeomorphic to some $\mathbb{Q}$ -algebraic set X' of $\mathbb{R}^m$, with m greater than or equal to n ?*

After motivating our choice of methods, highlighting their deep connections with recent advances in algebraic geometry over subfields developed by Fernando and Ghiloni, I will outline our positive answers to the Q-algebraicity problem. Our first main result is a version of the Nash-Tognoli theorem over $\mathbb{Q}$, which we can summarize as follows: *Every compact smooth manifold admits a $\mathbb{Q}$ -algebraic model.* As an application of resolution of singularities, a relative version of the Nash-Tognoli theorem over $\mathbb{Q}$, and blowing-down techniques, we provide a complete positive answer to the Q-algebraicity problem in the case of real algebraic sets with isolated singularities.

This is a joint work with Riccardo Ghiloni.

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