
Thom-Milnor bounds for smooth manifolds

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Abstract

How complicated can the zero set of a smooth map be once we leave the algebraic world? The guiding idea of this talk is that topological complexity is governed by distance from singularity in function space. On a compact Riemannian manifold, consider the discriminant—the set of maps that admit a singular zero—sitting inside the infinite-dimensional space of C^1 maps. We measure how far a given map is from this discriminant (a quantity computable from its first jet), and use it to define a “condition number” (the ratio between the C^1 norm and the distance to the discriminant). The main theorem shows that the total Betti number of the zero set is bounded by a polynomial in this condition number, with a sharp exponent equal to the dimension of the manifold. In plain terms: topology can proliferate only as a map approaches the discriminant, but this happens only at a rate limited by dimension.

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