Sard Properties and Asymptotic Infinite-Dimensional Real Algebraic Geometry

Antonio Lerario*[1]

[1]Scuola Internazionale Superiore di Studi Avanzati – Italy

Abstract

Sard's theorem asserts that the set of critical values of a smooth map between Euclidean spaces has measure zero. Smale extended this to infinite-dimensional Banach manifolds under the Fredholm condition, but beyond this framework the situation is poorly understood: when the domain is infinite dimensional and the target finite dimensional, the conclusion can fail even for polynomial maps! In this talk I will report on recent work where we provide sharp quantitative criteria ensuring Sard-type properties in this setting. Our approach combines tools from quantitative semialgebraic geometry, most notably a new approximate definable choice theorem, with complexity estimates in terms of Vitushkin variations and Kolmogorov n-widths. This framework can be viewed as a form of asymptotic infinite-dimensional real algebraic geometry, where the key parameter is not degree but the number of variables, and where novel dimension-independent bounds replace the usual large-degree asymptotics. As an application, we establish Sard-type results for Endpoint maps of Carnot groups restricted to piecewise real-analytic or entire controls.
Based on joint work with L. Rizzi and D. Tiberio

*Speaker