Totally real divisors on curves

Lorenzo Baldi*[1], Mario Kummer[2], and Daniel Plaumann[3]

[1]Universität Leipzig – Germany
[2]TU Dresden – Germany
[3]TU Dortmund – Germany

Abstract

Motivated by the study of nonnegative polynomials and of the dual moment problem, we study effective totally real divisors on a curve X defined over a real closed field, i.e., effective divisors supported on the real locus. Scheiderer proved that, for smooth curves over the real numbers with nonempty real locus, each divisor of sufficiently high degree is linearly equivalent to an effective totally real one. The smallest degree N(X) with this property is called the totally real divisor threshold.

When the field is non-Archimedean, we obtain a classification of topological types of smooth curves for which N(X) can be infinite. As a consequence, for curves over the real numbers, we prove that N(X) cannot be bounded from above only in terms of the topological type, unless the real locus has many connected components. We complement this qualitative result with a quantitative lower bound for N(X), depending on metric properties of the Jacobian and the curve in the Bergman metric.
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*Speaker