Real Algebraic Geometry and Interactions. Schedule

Monday, October 6

14:00 - 15:00 **Adam Parusiński**. *Zariski's dimensionality type of singularities. Case of dimensionality type 2.*
15:00 - 16:00 **Enrico Savi**. *The Nash-Tognoli theorem over the rational numbers and its version for isolated singularities*
16:00 - 16:30 *Coffee break*
16:30 - 17:30 **Lorenzo Baldi**. *Totally real divisors on curves*

Tuesday, October 7

9:30 - 10:30 **Erwan Brugallé**. *Welschinger-Witt invariants*
10:30 - 11:00 *Coffee break*
11:00 - 12:00 **Boulos El Hilany**. *Improved fewnomial upper bounds from Wronskians and dessins d'enfant*
12:00 - 14:00 *Lunch*
14:00 - 15:00 **Arthur Renaudineau**. *Patchworking and mirror symmetry*
15:00 - 16:00 **Laurent Busé**. *Discriminants of hypersurfaces with prescribed ordinary singularities*
16:00 - 16:30 *Coffee break*
16:30 - 17:30 **Jules Chenal**. *Prime Decomposition of Real Toric Threefolds*

Wednesday, October 8

9:30 - 10:30 **Lucas Gamertsfelder**. *Effective Countable Generalized Moment Problems*
10:30 - 11:00 *Coffee break*
11:00 - 12:00 **Giorgio Ottaviani**. *Persistent symmetric tensors*
12:00 - 14:00 *Lunch*
14:00 - 15:00 **Ilia Itenberg**. *Real plane sextic curves with smooth real part*
15:00 - 16:00 **Bruce Reznick**. *Higher Order Arithmetic-Geometric Inequalities*

Thursday, October 9

9:30 - 10:30 **Alicia Dickenstein**. *Real and sparse intersections*
10:30 - 11:00 *Coffee break*
11:00 - 12:00 **Frédéric Bihan**. *On multivariate generalizations of Descartes' rule of signs*
12:00 - 14:00 *Lunch*
14:00 - 15:00 **Jean-Yves Welschinger**. *Harnack hypersurfaces*
15:00 - 16:00 **Elias Tsigaridas**. *Positivity certificates for multivariate polynomials*
16:00 - 16:30 *Coffee break*
16:30 - 17:30 **Frédéric Mangolte**. *Real loci of rational Fano threefolds*

Friday, October 10

9:30 - 10:30 **Antonio Lerario**. *Thom-Milnor bounds for smooth manifolds*
10:30 - 11:00 *Coffee break*
11:00 - 12:00 **Viatcheslav Kharlamov**. *In a search of maximal real structures on punctual Hilbert schemes of surfaces*
12:00 - 14:00 *Lunch*

Real Algebraic Geometry and Interactions

Monday, October 6

14:00 - 15:00 **Adam Parusiński** (Université Côte d'Azur).

Zariski's dimensionality type of singularities. Case of dimensionality type 2.

In the 1970s O. Zariski introduced a general equisingularity theory for algebroid and algebraic hypersurfaces over an algebraically closed field of characteristic zero. His theory builds up on understanding the dimensionality type of hypersurface singularities, notion defined re- cursively by considering the discriminants loci of successive "generic" corank 1 projections. The theory of singularities of dimensionality type 1, that is the ones appearing generically in codimension 1, was developed by Zariski in his foundational papers on equisingular families of plane curve singularities. We give a similar complete description for singularities of dimen- sionality type 2, that is for families of surface singularities in the affine space of dimension three. Moreover, we show that in this case the generic linear projections are generic in the sense of Zariski, this is still an open problem for the dimensionality type greater than 2. (Joint work with Laurentiu Paunescu, the University of Sydney)

15:00 - 16:00 **Enrico Savi** (Université d'Angers).

The Nash-Tognoli theorem over the rational numbers and its version for isolated singularities

In this talk I will introduce and discuss several possible approaches to addressing the following open question:

$\mathbb{Q}$-algebraicity problem: (Parusinski, 2021) *Is every algebraic subset X of $\mathbb{R}^n$ homeomorphic to some $\mathbb{Q}$-algebraic set X' of $\mathbb{R}^m$, with m greater than or equal to n?*

After motivating our choice of methods, highlighting their deep connections with recent advances in algebraic geometry over subfields developed by Fernando and Ghiloni, I will outline our positive answers to the $\mathbb{Q}$-algebraicity problem. Our first main result is a version of the Nash-Tognoli theorem over $\mathbb{Q}$, which we can summarize as follows: *Every compact smooth manifold admits a $\mathbb{Q}$-algebraic model.* As an application of resolution of singularities, a relative version of the Nash-Tognoli theorem over $\mathbb{Q}$, and blowing-down techniques, we provide a complete positive answer to the $\mathbb{Q}$-algebraicity problem in the case of real algebraic sets with isolated singularities.

This is a joint work with Riccardo Ghiloni.

Real Algebraic Geometry and Interactions

Monday, October 6

16:30 - 17:30 **Lorenzo Baldi** (Universität Leipzig).

Totally real divisors on curves

Motivated by the study of nonnegative polynomials and of the dual moment problem, we study effective totally real divisors on a curve X defined over a real closed field, i.e., effective divisors supported on the real locus. Scheiderer proved that, for smooth curves over the real numbers with nonempty real locus, each divisor of sufficiently high degree is linearly equivalent to an effective totally real one. The smallest degree N(X) with this property is called the totally real divisor threshold.

When the field is non-Archimedean, we obtain a classification of topological types of smooth curves for which N(X) can be infinite. As a consequence, for curves over the real numbers, we prove that N(X) cannot be bounded from above only in terms of the topological type, unless the real locus has many connected components. We complement this qualitative result with a quantitative lower bound for N(X), depending on metric properties of the Jacobian and the curve in the Bergman metric. Based on the joint work https://www.arxiv.org/abs/2509.07544 with M. Kummer and D. Plaumann.

Real Algebraic Geometry and Interactions

Tuesday, October 7

9:30 - 10:30 **Erwan Brugallé** (Université de Nantes).

Welschinger-Witt invariants

This talk will address the problem of enumerating with quadratic forms rational curves in complex surfaces or symplectic 4-manifolds. The real Abramovich-Bertram formula for Welschinger invariants of real symplectic rational 4-manifolds allows one to encode them in what is known as a Witt invariant over any field. It turns out that these Weslchinger-Witt invariants recover the quadratic Gromov-Witten invariants, recently defined by Kass-Levine-Solomon-Wickelgren, in the case of rational del Pezzo surface of degree at teast 6 (and conjecturally of degree at least 3). As a consequence, quadratic Gromov-Witten invariants of these rational surfaces over any field are determined by the two special fields $\mathbb{C}$ and $\mathbb{R}$.

This is a joint work with Johannes Rau and Kirsten Wickelgren.

11:00 - 12:00 **Boulos El Hilany** (TU Braunschweig).

Improved fewnomial upper bounds from Wronskians and dessins d'enfant

This talk focuses on counting solutions to a real square polynomial system $f = g = 0$, where f has three monomial terms and g has t monomial terms. Over the past 25 years, ongoing efforts to find a sharp upper bound on the maximal number of such solutions in the real positive orthant have produced a variety of methods, each leading to incremental im- provements. Currently, the best-known upper bound is a cubic polynomial in t.

I will highlight some of the methods used for this fewnomial problem, and focus on the latest iteration that utilizes a combination of Wronskians and Grothendieck's dessins d'enfant. The talk will conclude with some generalization strategies and related open problems.

This is a joint work with Sébastien Tavenas.

Real Algebraic Geometry and Interactions

Tuesday, October 7

14:00 - 15:00 **Arthur Renaudineau** (Université de Lille).

Patchworking and mirror symmetry

In this talk, I will discuss a special case of combinatorial patchworking for primitive central triangulations of reflexive polytopes. This gives examples of real Calabi-Yau hypersurfaces in Fano toric varieties. I will show how to use a combinatorial analogue of mirror symmetry to deduce some informations on the real hypersurface and give examples.

15:00 - 16:00 **Laurent Busé** (INRIA Sophia Antipolis).

Discriminants of hypersurfaces with prescribed ordinary singularities

Let h be the generic form of a hypersurface in projective n-space with at most $n+1$ isolated ordinary singularities in general position. Using weight properties of the coefficients of h, we will present a formula relating the component of lowest weight of the classical discriminant of a perturbation of the form $h + tg$ (where t is a new variable and g is a general form) to the sparse discriminant of h. The proof of this formula also provides an alternative approach to sparse resultants and discriminants in our specific context, which we will discuss. This is joint work with Thomas Dedieu.

16:30 - 17:30 **Jules Chenal** (Universitetet i Oslo).

Prime Decomposition of Real Toric Threefolds

A differential threefold is prime when it does not admit a non-trivial connected sum decomposition. A theorem of Kneser and Milnor asserts that every threefold can be uniquely decomposed as the connected sum of such prime manifolds. In this talk, we will provide the prime decomposition of the real loci of smooth toric threefolds. Usually a toric threefold is understood as acted upon by the cube of the real multiplicative group, i.e. the split tridimensional torus. However, there are six different tridimensional tori. First, we will recall the results of Erokhovets for toric threefolds under the action of the split torus. Then, we will focus on the five other cases. This is based on a joint work with Matilde Manzaroli.

Real Algebraic Geometry and Interactions

Wednesday, October 8

9:30 - 10:30 **Lucas Gamertsfelder** (Inria Sophia Antipolis).

Effective Countable Generalized Moment Problems

The Generalized Moment Problem (GMP) provides a unified framework for optimization problems that can be formulated as linear programs over measures. Examples of such prob- lems include polynomial optimization, optimal control, and symmetric tensor decomposition. To solve these infinite-dimensional problems, Moment-Sum-of-Squares (SoS) hierarchies of- fer a sequence of finite-dimensional semidefinite relaxations. These hierarchies are known to converge under standard (Archimedean and compactness) conditions.

Recently, progress has been focused on the convergence rate of these hierarchies, with rate analyses now available for polynomial optimization, volume computation, and optimal control. We build on these advances by establishing convergence rates for the GMP itself.

Under standard Archimedean, S-fullness, and dual attainment conditions, we provide new geometry-adaptive bounds for problems with countable moment constraints on vectors of measures. These results guarantee convergence for the optimal values, the feasibility sets in Hausdorff distance, and the optimizers in the weak* topology. We demonstrate our results by providing new, effective convergence rates for symmetric tensor decomposition.

11:00 - 12:00 **Giorgio Ottaviani** (Università di Firenze).

Persistent symmetric tensors

Persistent tensors were introduced by Gharahi and Lysikov via a recursive construction inspired by quantum information theory. In a nutshell, they constitute a class of tensors that remain well-behaved under the substitution method and, consequently, admit nontrivial lower bounds on tensor rank. In a work joint with Gharahi, we investigate the symmetric case, namely, symmetric persistent tensors, or equivalently, persistent polynomials. We establish that a symmetric tensor is persistent if the determinant of its Hessian equals the power of a nonzero linear form. The converse is verified for cubic tensors and in small dimension, by leveraging classical results of B. Segre. We discuss connections with invariants of some prehomogeneous spaces, homaloidal polynomials, and Legendre transforms.

Real Algebraic Geometry and Interactions

Wednesday, October 8

14:00 - 15:00 **Ilia Itenberg** (IMJ-PRG – Sorbonne Université).

Real plane sextic curves with smooth real part

The talk is devoted to the curves of degree 6 in the real projective plane. We show that the equisingular deformation type of a simple real plane sextic curve with smooth real part is determined by its real homological type, that is, the polarization, excep- tional divisors, and real structure recorded in the homology of the covering K3-surface. We will also present an Arnold-Gudkov-Rokhlin type congruence for real algebraic curves/surfaces with certain singularities and a result concerning contraction of ovals of a singular real plane sextic with smooth real part. (This is a joint work with Alex Degtyarev.)

15:00 - 16:00 **Bruce Reznick** (University of Illinois).

Higher Order Arithmetic-Geometric Inequalities

If $\{\alpha_k\} \subset \mathbb{R}^d$ consists of a simplex and a single interior point, and if you impose the condition that $p(x) = \sum c_k x^{\alpha_k}$ vanishes to the second order at $\underline{1} = (1, \ldots, 1)$, then the resulting polynomial is, up to a multiple, a version of the arithmetic-geometric inequality for the monomials $\{x^{\alpha_k}\}$. In this talk, we explore geometric conditions on larger point-sets $\{\alpha_k\}$ so that imposing a higher even-order vanishing at $\underline{1}$ leads to an inequality, and present a few preliminary results and a lot of pictures.

Real Algebraic Geometry and Interactions

Thursday, October 9

9:30 - 10:30 **Alicia Dickenstein** (University of Buenos Aires).

Real and sparse intersections

I will describe some lower and upper bounds on the number of positive solutions to parametrized systems of polynomial equations with fixed supports, inspired by applications, and pose some basic open problems. If time permits, I will present a sharp upper bound, obtained in collaboration with Ezequiel Fattori, for the number of positive solutions to a family of systems arising from the study of enzymatic mechanisms.

11:00 - 12:00 **Frédéric Bihan** (Université de Savoie).

About multivariate generalizations of Descartes'rule of signs

In this talk, we will describe generalizations of Descartes' rule in the case of multiple variables which were obtained recently, as well as research in progress on the subject.

14:00 - 15:00 **Jean-Yves Welschinger** (CNRS / Université Lyon 1).

Harnack hypersurfaces

I will discuss a higher dimensional counterpart to the method of construction introduced by Axel Harnack, which produces smooth real hypersurfaces in projective spaces. I will then investigate the topology of the latter, which turns out to be partially governed by an equation and will finally solve this equation, yielding to "universal Harnack hypersurfaces". This is a joint work - in progress - with Michele Ancona and Erwan Brugallé.

15:00 - 16:00 **Elias Tsigaridas** (Inria Paris).

Positivity certificates for multivariate polynomials

We present mathematical, algorithmic, and complexity results on defining and computing SOS certificates of positivity for (multivariate) polynomials (in $n \geq 1$ variables) with rational coefficients, that are positive over $\mathbb{R}^n$.

Joint work(s) with Matias Bender, Chaoping Zhu, and Khazhgali Kozhasov.

16:30 - 17:30 **Frédéric Mangolte** (Aix-Marseille Université).

Real loci of rational Fano threefolds

From the classification of real rational surfaces worked out by Comessatti at the beginning of the 20th century we get the following striking characterization of real rational surfaces: a geometrically rational real surface is rational if and only if its real locus is non-empty and connected. The analogous assertion fails in higher dimension.
With Andrea Fanelli, we explore real loci of geometrically rational Fano threefolds in relation to their rationality and find an unexpected criterion.

https://arxiv.org/abs/2507.04012

Real Algebraic Geometry and Interactions

Friday, October 10

9:30 - 10:30 **Antonio Lerario** (SISSA).

Thom-Milnor bounds for smooth manifolds

How complicated can the zero set of a smooth map be once we leave the algebraic world? The guiding idea of this talk is that topological complexity is governed by distance from singularity in function space. On a compact Riemannian manifold, consider the discriminant—the set of maps that admit a singular zero—sitting inside the infinite-dimensional space of C^1 maps. We measure how far a given map is from this discriminant (a quantity computable from its first jet), and use it to define a "condition number" (the ratio between the C^1 norm and the distance to the discriminant). The main theorem shows that the total Betti number of the zero set is bounded by a polynomial in this condition number, with a sharp exponent equal to the dimension of the manifold. In plain terms: topology can proliferate only as a map approaches the discriminant, but this happens only at a rate limited by dimension.

11:00 - 12:00 **Viatcheslav Kharlamov** (Université de Strasbourg).

In a search of maximal real structures on punctual Hilbert schemes of surfaces

In a recent joint work with R. Rasdeaconu, we investigated the maximality of the Hilbert square of real algebraic varieties. We observed that, starting from dimension two, many deformation classes of algebraic varieties do not contain any real variety whose Hilbert square is maximal. In particular, we found that the K3-surfaces have never a maximal Hilbert square. In this talk I will speak on generalizations of these results to Hilbert schemes of n points for any n as well as to hyperkähler varieties of $K3^{[n]}$-type. The talk is based on a work in progress with Simone Billi, Lie Fu, and Annalisa Grossi.